Problem of the Week
Problem A and Solution
Sweet Possibilities

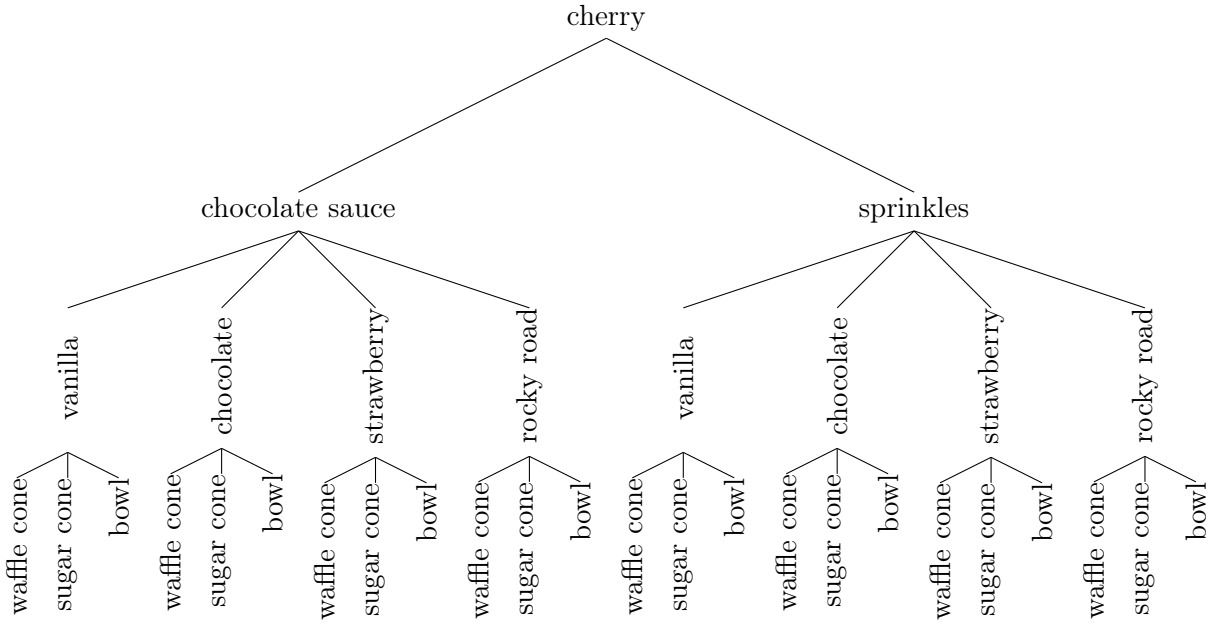
Problem

At Ahmed's birthday party, his guests can build their own dessert. They start by choosing either a waffle cone, sugar cone, or a bowl. Then they choose from the following ice cream flavours: vanilla, chocolate, strawberry and rocky road. Next they add either chocolate sauce or sprinkles. Finally, they can add a cherry on top if they wish.

How many different dessert combinations are possible? Justify your answer.

Solution

This tree shows all of the possible combinations of ice cream treats that include a cherry.



There are a total of 24 combinations. You could make a similar tree for the 24 combinations of ice cream without a cherry. This means that there are a total of 48 dessert combinations possible.

Note that this is not the only tree that shows all possible combinations. You could also build a tree that starts with the type of cone and works down to adding a cherry or not. In fact, you could have the choices appear in any order you like within the levels of the tree. However, all of these trees would show that there are a total of 48 combinations.

Teacher's Notes

We can calculate the number of combinations in a more formulaic way. Suppose we had just two choices to make: the base of the dessert and the type of ice cream. For each of the three bases there are four types of ice cream that can fill them. In other words, we multiply the number of choices we have in each case to determine the number of combinations. This means that there would be $3 \times 4 = 12$ different combinations of bases and ice cream. For this problem there are four sets of choices to make. This means the number of combinations can be calculated as:

$$\underset{\text{choices for the base}}{3} \times \underset{\text{choices of ice cream}}{4} \times \underset{\text{choices of topping}}{2} \times \underset{\text{cherry options}}{2} = 48 \text{ combinations}$$

The tree gives more information than just the number of combinations. The tree also shows the actual arrangements of the dessert. For example, one combination has a *cherry* on top of *chocolate sauce* on *vanilla ice cream* in a *bowl*. You can see that combination by following a **path** that starts at the top of the tree, known as the **root**, moves down one level on each step, and ends at the bowl under vanilla. There is a unique path from the top of the tree to each of the **nodes** at the bottom level of the tree.

The properties and uses of trees are studied in an area of mathematics known as graph theory. Trees are also very useful structures used in many computer programs to solve a variety of problems.